

Basic Mathematics Assignment Grade XI(2079 Kartik 28)

Complex number

1. If $x + iy = \frac{3}{2 + \cos\theta + i\sin\theta}$, show that $x^2 + y^2 = 4x - 3$
2. Show that $\frac{3+2i}{2-5i} + \frac{3-2i}{2+5i}$ is purely a real number.
3. If z and w are complex numbers, show that $|z| + |w| \geq |z + w|$
4. Determine the square roots of i) $-8 + 6i$ ii) i iii) $\frac{2-5i}{6+i}$
5. If $(x - iy)^{\frac{1}{3}} = a - ib$, show that $\frac{x}{a} + \frac{y}{b} = 4(a^2 - b^2)$

Quadratic Equation

6. If a, b, c are rationals and $a + b + c = 0$, show that the roots of $(b + c - a)x^2 + (c + a - b)x + (a + b - c) = 0$ are rational.
7. If α and β are the roots of the equation $2x^2 - 3x - 5 = 0$, form a quadratic equation whose roots are $2\alpha + \frac{1}{\beta}$ and $2\beta + \frac{1}{\alpha}$
8. Prove that if the equations $x^2 + bx + c = 0$ and $x^2 + cx + ab = 0$ have a common root, their other roots will satisfy $x^2 + ax + bc = 0$.
9. Determine the values of p for which the equations $3x^2 + 4px + 2 = 0$ and $2x^2 + 3x - 2 = 0$ may have a common root. (Ans 56, -44)
10. If the equation $x^2 + 2(k + 2)x + 9k = 0$ has equal roots, find k .

Analytical Geometry

11. Find the equations of the bisectors of the angles between $4x - 3y + 1 = 0$ and $12x - 5y + 7 = 0$. Prove that the bisectors are at right angle to each other.
12. If p and p' be the length of the perpendiculars from the origin upon the straight line whose equations are $x \sec\theta + y \operatorname{cosec}\theta = a$ and $x \cos\theta - y \sin\theta = a \cos 2\theta$, prove that $4p^2 + p'^2 = a^2$.
13. Find the equation to the pair of straight lines joining the origin to the intersection of the straight line $y = mx + c$ and the curve $x^2 + y^2 = a^2$. Prove that they are at right angle if $2c^2 = a^2(1 + m^2)$
14. For what value of k does the equation $12x^2 - 10xy + 2y^2 + 11x - 5y + k = 0$ represent two straight lines? Show that the angle between them is $\tan^{-1}\left(\frac{1}{7}\right)$.

15. Find the equation to the straight lines passing through (1,1) and parallel to the lines represented by $x^2 - 5xy + 4y^2 + x + 2y - 2 = 0$

Trogonometry , Limit and Continuity

16. If $\cos^{-1}x + \cos^{-1}y + \cos^{-1}z = \pi$, show that $x^2 + y^2 + z^2 + 2xyz = 1$

17. Prove that $\tan\left(\frac{1}{2}\sin^{-1}\frac{2x}{1+x^2} + \frac{1}{2}\cos^{-1}\frac{1-y^2}{1+y^2}\right) = \frac{x+y}{1-xy}$

18. Evaluate: i) $\lim_{x \rightarrow 1} \frac{x - \sqrt{2-x^2}}{2x - \sqrt{2+2x^2}}$ ii) $\lim_{x \rightarrow \theta} \frac{x \tan \theta - \theta \tan x}{x - \theta}$

iii) $\lim_{y \rightarrow 0} \frac{(x+y) \sec(x+y) - x \sec x}{y}$ iv) $\lim_{x \rightarrow \frac{\pi}{2}} (a - x) \tan \frac{\pi x}{2a}$

19. A function $f(x)$ is defined as $f(x) = \begin{cases} 2x - 3 & \text{for } x < 2 \\ 2 & \text{for } x = 2 \\ 3x - 5 & \text{for } x > 2 \end{cases}$. Is the function

$f(x)$ continuous at $x = 2$? If not, how can the function $f(x)$ be made continuous at $x = 2$?

20. i) Find the points of discontinuity of the function defined by $f(x) = \frac{x^2 - 4}{x^2 - 5x + 6}$.

ii) Find the left hand limit, right hand limit and functional value of the function defined by $f(x) = \frac{|x-2|}{x-2}$ at $x = 2$. Also comment on the continuity of the function.