

## Class XII Assignment of Basic Mathematics

### Permutation Combination

1. Show that the number of ways in which the letters of the word “ARRANGE” can be arranged so that no two R’s come together is 900.
2. Show that the number of ways in which the letters of the word “CALCULUS” can be arranged so that two C’s don't come together is 3780
3. In how many ways can eight people be seated in a round table if two people insist in sitting next to each other?
4. From 6 gentlemen and 4 ladies, a committee of 5 is to be formed. In how many ways this be done so as to include at least on lady?
5. In a group of 10 students, 6 are boys. In how many ways can 4 students be selected for mathematical competition so as to include at least two boys?

### Binomial Theorem

6. Show that  $1 + \frac{1+2}{2!} + \frac{1+2+3}{3!} + \frac{1+2+3+4}{4!} + \dots$  to  $\infty = \frac{3e}{2}$

7. Using Binomial theorem derive the expansion of  $e^x$ .

8. If  $(1 + x)^n = C_0 + C_1x + C_2x^2 + \dots + C_nx^n$ ,

prove that  $C_0 + 3C_1 + 5C_2 + \dots + (2n + 1)C_n = (n+1)2^n$

9. Find the 5<sup>th</sup> term in the binomial expansion of  $\left(x - \frac{2}{x}\right)^7$

10) Show that  $\sum_{n=1}^{\infty} \frac{n^2}{(n+1)!} = e - 1$

### System of linear equation and conic section

11) Solve the system of equations  $x + 2y - z = -5$ ;  $2x - y + z = 6$ ;  $x - y - 3z = -3$   
by row equivalent matrix method .

12) Solve the system of equations  $x + 4y + 3z = 6$ ;  $3x + 9y = 18$ ;  $-5x - 6y + 2z = -5$   
by inverse matrix method.

13) Find the eccentricity, vertex and foci of the ellipse  $\frac{(x+2)^2}{16} + \frac{(y-5)^2}{9} = 1$

14) Find the eccentricity, foci and vertices of the hyperbola  $9x^2 - 16y^2 - 18x - 64y - 199 = 0$ .

15) Classify the system of equations  $3x - 4y = 1$ ;  $6x - 8y = 7$

### Calculus

16. Find, from the first principle, the derivative of  $f(x) = \log(\tan x)$ .

17) If  $x$  changes from 4 to 4.1, then find the exact change and approximate change in the function  $x^2 + x$  at  $x = 4$ .

18) Using L Hospital's rule, evaluate: i)  $\lim_{x \rightarrow 0} \frac{\log \sin x}{\cot x}$  ii)  $\lim_{x \rightarrow 0} \frac{\tan x - x}{x - \sin x}$

19) State Rolle's theorem. Verify this theorem for the function  $f(x) = (x + 1)(x - 2)$  in  $[-1, 2]$ .

20) Find  $\frac{dy}{dx}$  i)  $\left(\sinh \frac{x}{a} + \cosh \frac{x}{a}\right)^{nx}$  ii)  $y = 2 \tan^{-1}\left(\tanh \frac{x}{2}\right)$

### Mathematics for economics and finance

21) The demand function for a good is  $P = 24 - 2Q$ . Fixed cost of the goods is Rs 30 and the variable cost for each additional unit of good is Rs 8.

- Express TC and TR in terms of  $Q$ .
- Find break- even point.
- Find the maximum revenue.

22). The following table shows the inter-relationship between the product of two industries A and B in a year

Industry	A	B	Consumer's demand	Total output
A	45	50	55	150
B	30	40	30	150

- Find the input coefficient matrix
- Find the gross output of the two industries A and B to satisfy the demands of 72 and 48 units.

23). In a competitive market, the demand and the supply function of fruits are

$Q_d = 240 - 1.4P$  and  $Q_s = -60 + 2.6P$  and the rate of price adjustment proportional to the excess of demand is given by  $\frac{dP}{dt} = 0.2(Q_d - Q_s)$  with the initial price level of Rs 120. What will be  $P(t)$  when  $t = 3$ . In what time period would its price level of Rs 84?

24) Find the producer surplus for the supply function  $P = 3 + 2Q$  at  $P = 33$ .

25) The population of a certain city increases at the rate of 2% p.a. If the initial population of the city in the year 2010 be 1,50,000, what was the population of the city in 2020?

### Trigonometry and vector

26) Which one of the following is the domain of the function  $y = \sin^{-1} \frac{x}{3}$

- i)  $-1 \leq x \leq 1$     ii)  $-3 \leq x \leq 3$     iii)  $-\infty \leq x \leq \infty$     iv)  $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$

27) Prove that  $\tan^{-1} \sqrt{x} = \frac{1}{2} \cos^{-1} \left( \frac{1-x}{1+x} \right) = \frac{1}{2} \sin^{-1} \frac{2\sqrt{x}}{1+x}$

28) If  $\tan^{-1} \frac{\sqrt{1+x^2} - \sqrt{1-x^2}}{\sqrt{1+x^2} + \sqrt{1-x^2}} = \alpha$ , prove that  $x^2 = \sin 2\alpha$ . (Hint: put  $x^2 = \cos \theta$  and proceed)

29) Using the vector method, prove that  $\sin(A + B) = \sin A \cos B + \cos A \sin B$ .

30) Obtain the sides, angles and the area of the triangle formed by the points whose position vectors are  $2\vec{i} - \vec{j} + 3\vec{k}$ ,  $\vec{i} + 2\vec{j} - 3\vec{k}$  and  $\vec{i} - \vec{j} - 2\vec{k}$