| The Times Secondary School |  |  |
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| Grade: - XII | Set A | Full Marks:-75 |
| Stream: Science |  | Pass Marks:-30 |
| Subject: - Mathematics |  | Time: 3hrs |

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate the full marks.

Group-A $[11 \times 1=11]$

## Rewrite the correct option in your answer sheet.

1. How many odd numbers of three different digits can be formed from the integers $1,2,3$, 4 and 5?
a) 12
b) 36
c) 60
d) 120
2. The middle term in the expansion of $\left(x-\frac{1}{x}\right)^{20}$ is
a) $\mathrm{C}(20,10)$
b) - $\mathrm{C}(20,10)$
c) $C(20,10) x$
d) $C(20,10) x^{10}$
3. Sum of the first $n$ natural numbers is
a) $n^{2}$
b) $\frac{n(n+1)(2 n+1)}{}$
c) $\frac{n(n+1)}{2}$
d) $\left(\frac{n(n+1)}{2}\right)^{2}$
4. The radius of the circle $x^{2}+y^{6}+4 x-6 y+4=0$ is
a) 2
b) 3
c) 4
d) 9
5. For what value of $p$, do the system of equations $2 x+3 y=1,4 x+6 y=p$ have infinite solutions?
a) 0
b) 1
c) 2
d) 4
6. $\lim _{x \rightarrow \frac{\pi}{2}} \frac{\tan 5 x}{\tan x}$ equals
a) $\frac{1}{5}$
b) $-\frac{1}{5}$
c) $\frac{5}{7}$
d) $-\frac{1}{7}$
7. The derivative of $\operatorname{Arc} \tan \sinh x$ is
a) $\cosh x$
b) $\sinh x$
c) $\tanh x$
d) $\operatorname{sech} x$
8. The integral value of $\int \frac{d x}{a^{2}-x^{2}}$ is
a) $\frac{1}{2 a} \ln \left(\frac{a+x}{a-x}\right)+\mathrm{c}$
b) $\frac{1}{a} \tan ^{-1} \frac{x}{a}+c$
c) $\log \left(x+\sqrt{a^{2}-x^{2}}\right)+c$
d) $\sin ^{-1} \frac{x}{a}+c$
9. If the regression coefficients are $\mathrm{b}_{\mathrm{xy}}=-\frac{1}{3}$ and $\mathrm{b}_{\mathrm{yx}}=-\frac{3}{4}$, the correlation coefficient between two vriables x and y is
a) 0.25
b) -.0 .25
c) 0.50
d) -0.50
10. If $\alpha$ and $\beta$ be two complex roots of unity then $\alpha^{2}$ equals
a) 1
b) 0
c) $\alpha$
d) $\beta$
11. For two dependent events $A$ and $B, P(A / B)$ equals
a) $\frac{P(A \cap B)}{P(A)}$
b) $P(A) P(B)$
c) $\frac{P(A \cap B)}{P(B)}$
d) $\frac{P(A)}{P(A \cap B)}$

## Group ' $\mathbf{B}$ ' $[\mathbf{5} \times \mathbf{8}=\mathbf{4 0}$ ]

12. (a) In how many ways can the letters of the word 'INTERVAL' be arranged so that the vowels may occupy only the odd positions?
b) A candidate has to pass in each of the five subjects. In how many ways can he be fail?
13. If $(1+x)^{n}=C_{0}+C_{1} x+C_{2} x^{2}+\ldots \ldots \ldots+C_{n} x^{n}$,
a) Write the value of
i) $\mathrm{C}_{0}+\mathrm{C}_{1}+\mathrm{C}_{2}+\mathrm{C}_{3}+\ldots \ldots .+\mathrm{C}_{\mathrm{n}}$
ii) $\mathrm{C}_{1}+2 \mathrm{C}_{2}+3 \mathrm{C}_{3}+\ldots \ldots . .+\mathrm{nC}_{n}$
b) Prove that. $\mathrm{C}_{0}+4 \mathrm{C}_{1}+7 \mathrm{C}_{2}+10 \mathrm{C}_{3}+\ldots \ldots .+(3 n+1) \mathrm{C}_{\mathrm{n}}=(3 \mathrm{n}+2) 2^{\mathrm{n}-1}$
14. Solve the system of equations: $x-y+2 z=0, \quad x-2 y+3 z=-1,2 x-2 y+z=-3$ by Row-equivalent matrix method or Cramar's Rule.
15. Find the regression equation of x on y from the following data from the following data

| X | 5 | 9 | 13 | 17 | 21 |
| :--- | :--- | :--- | :--- | :--- | :--- |
| Y | 3 | 8 | 13 | 18 | 23 |

Estimate th value of x when $\mathrm{y}=18$.
16. Solve the following LP Problem, using simplex method

Maximize $\mathrm{Z}=7 \mathrm{x}+5 \mathrm{y}$ subject to $x+2 y \leq 6,4 x+3 y \leq 12, x, y \geq 0$.
17.i) Define L'Hospital's Rule and use it to evaluate $\lim _{x \rightarrow \theta} \frac{x \sin \theta-\theta \sin x}{x-\theta}$
ii) Find the derivative of $\tanh x^{\operatorname{sech} x}$
18. a) Write the equation of tangent to the curve $\mathrm{y}=\mathrm{f}(\mathrm{x})$ at piont $\left(\mathrm{x}_{1}, \mathrm{y}_{1}\right)$ in differential form.
b) Define the angle of intersections between two curves.
b) Find the point on the curve $y=2 x^{2}+3 x+5$, the tangent at which is perpendicular to the line $\mathrm{x}-5 \mathrm{y}+10=0$.
19. Two concentric circles are expanding in such a way that the radius of inner circle is increasing at a rate of $4 \mathrm{~cm} / \mathrm{sec}$ and that of the outer circle at a rate of $1.5 \mathrm{~cm} / \mathrm{sec}$. Is the area between the circles increasing or decreasing when the inner and outer circles are 5 cm and 8 cm . And how fast?

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\begin{equation*}
\text { Group ' } C^{\prime}[8 \times 3=24] \tag{2}
\end{equation*}
$$

20. (a) Prove that $\frac{1}{2.3}+\frac{1}{4.5}+\frac{1}{6.7}+\ldots \ldots \ldots=1-\log _{\mathrm{e}} 2$
b) Using De Morgan's theorem find the value of $(1-\mathrm{i} \sqrt{3})^{6}$.
c) Using mathematical induction, prove that $1+3+5+\ldots+(2 n-1)=n^{2}$
21. a) Find the equation of tangent to the circle $x^{2}+y^{2}-6 x+8 y-4=0$ at $(8,6)$.
b) Find the condition that the line $l x+m y+n=0$ is tangent to the circle $x^{2}+y^{2}+2 g x+$ $2 f y+c=0$.
c) Find the equation of parabola whose vertex is at $(3,2)$ and the focus is at $(5,2)$.
22. a)Define integral of the function $f(x)$ w. r. t. $x$
b) Write the integral of $\int e^{a x} \sin b x d x$
c) Integrate the following
i) $\int \frac{d x}{1+\sin x+\cos x}$
ii) $\int \frac{x+1}{(x-4)(x+2)} d x$

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| Dillibazar, Kathmandu |  |

Candidates are required to give their answers in their own words as far as practicable. The figures in the margin indicate the full marks.

## Attempt all the questions.

## Group-A $[11 \times 1=11]$

## Rewrite the correct option in your answer sheet.

1. Of the numbers formed by using all the figures $1,2,3,4,5$ only once, how many are even?
a) 24
b) 36
c) 48
d) 96
2. The sum of the binomial coefficients in the expansion of $(1+x)^{n}$ is
a) 2 n
b) $2^{n}$
c) $n$ !
d) $n^{2}$
3. Sum of the squares of the first n natural numbers is
a) $\mathrm{n}^{2}$
b) $\frac{n(n+1)(2 n+1)}{6}$
c) $\frac{n(n+1)}{2}$
$\mathrm{d}\left(\frac{n(n+1)}{2}\right)^{2}$
4. The center of the circle $x^{2}+y^{2}+4{ }_{x}^{6}-6 y+4=0$ is
a) $(-2,-3)$
b) $(2,-3)$
c) $(-2,3)$
d) $(2,3)$
5. The system of equations $x-2 y=5$ and $k x+6 y=9$ have no solutions when $k$ equals
a) -2
b) 2
c) -3
d) 3
6. $\lim _{x \rightarrow \frac{\pi}{2}} \frac{\sec 7 x}{\sec 5 x}$ equals
a) $\frac{7}{5}$
b) $-\frac{7}{5}$
c) $\frac{5}{7}$
d) $-\frac{5}{7}$
7. The derivative of $\log (\tanh x)$ is
c) $2 \operatorname{cosech} 2 x$
d) $\operatorname{sech}^{2} x$
8. The integral of $\int \frac{d x}{\sqrt{x^{2}+a^{2}}}$ is
a) $\log \left(x+\sqrt{x^{2}-a^{2}}\right)+c$
b) $\log \left(x-\sqrt{x^{2}-a^{2}}\right)+c$
c) $\log \left(x+\sqrt{x^{2}+a^{2}}\right)+c$
d) $\sin ^{-1} \frac{x}{a}+c$
9. The correlation coefficinet ' $r$ ' between two variables lies between
a) $-\infty \leq r \leq \infty$
b) $-1 \leq r \leq 1$
c) $0 \leq r \leq 1$
d) $0 \leq r \leq \infty$
10. If $\alpha$ and $\beta$ be two complex roots of unity then $\alpha^{-1}$ equals
a) 1
b) 0
c) $\alpha$
d ) $\beta$
11. For two dependent events $A$ and $B, P(B / A)$ equals
a) $\frac{P(A \cap B)}{P(A)}$
b). $P(A) P(B)$
c). $\frac{P(A \cap B)}{P(B)}$
d) $\frac{P(A)}{P(A \cap B)}$

Group ' $B$ ' $[5 \times 8=40]$
12. (a) In how many ways can the letters of the word "CALCULUS" be arranged so that two C's do not come together. [3]
b) How many committees of 3 men and 2 women can be formed from 12 men and 8 women? [2]
13. a) Define Eulers number. [1]
b) Show that $1+\frac{1+2}{2!}+\frac{1+2+3}{3!}+\frac{1+2+3+4}{4!}+\ldots \ldots \ldots$ to $\infty=\frac{3 e}{2}$.
14. Solve the system of equatiions: $x-y+z=-3, \quad x+y+z=1,3 x-4 y-z=1$
by Row-equivalent matrix method or Cramars Rule.
15. Find the coefficient of correlation by Karl Pearson's method.

| X | 6 | 2 | 10 | 4 | 8 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Y | 9 | 11 | 6 | 8 | 7 |

16. Solve the following LP Problem, using simplex method

Maximize $Z=5 x-3 y$ subject to $3 x+2 y \leq 6, x-3 y \leq 4, x, y \geq 0$
17. a)Define L'Hospital's Rule and use it to evaluate $\lim _{x \rightarrow \theta} \frac{\tan b x}{\operatorname{tancx}}$
b) Find the derivative of $\sinh x \cosh x$
18. a) Write the equation of normal to the curve $y=f(x)$ at piont $\left(x_{1}, y_{1}\right)$ in differential form.
b) Define the angle of intersections between two curves.
c) Find the point on the curve $2 y=3-x^{2,}$ the tangent at which is parallel to the line $x+y=0$
19. An inverted cone has depth of 40 cm and a base of radius 5 cm . Water is poured into it at a rate of 1.5 cubic centimeters per second. Find the rate at which the level of water in the cone is rising when the depth is 4 cm .

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\begin{equation*}
\text { Group ' } C \text { ' }[8 \times 3=24] \tag{3}
\end{equation*}
$$

20. (a) If the coefficient of $x$ in the expansion of $\left(x^{2}+\frac{k}{x}\right)^{5}$ is 270 find $k$.
(b) Using De Morgan's theorem find the value of $(1+i)^{20}$.
c) Using mathematical induction, prove that $2+4+6+\ldots+2 n=n(n+1)$
21. a) Find the equation of tangents to the circle $x^{2}+y^{2}-2 x-4 y+3=0$ at $(2,3)$.
b) Prove that the straight line $y=x+a \sqrt{2} x$ touches the circle $x^{2}+y^{2}=a^{2}$. Also, find the point of contact.
c) Find the equation of parabola whose vertex is at $(5,3)$ and the focus is at $(5,6)$.
22. a)Define integral of the function $f(x)$ w. r. t. $x$
b) Write the integral of $\int \sqrt{x^{2}+a^{2}} d x$
c) Integrate the following

$$
\begin{array}{ll}
\text { i) } \int \frac{d x}{\sin x+\cos x} & \text { ii) } \int \frac{x-1}{(x-2)(x+1)} d x
\end{array}
$$

